Is the dark halo of our Galaxy spherical?

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ABSTRACT

It has been recently claimed that the confined structure of the debris from the Sagittarius dwarf implies that the dark-matter halo of our Galaxy should be nearly spherical, in strong contrast with predictions from cold dark-matter (CDM) simulations, where dark halos are found to have typical density axis ratios of 0.6-0.8. In this letter, numerical simulations are used to show that the Sagittarius streams discovered thus far are too young dynamically to be sensitive to the shape of the dark halo of the Milky Way. The data presently available are entirely consistent with a Galactic dark-matter halo that could either be oblate or prolate, with minor-to-major density axis ratios c/a as low as 0.6 within the region probed by the orbit of the Sagittarius dwarf.

Key words: dark-matter - galaxies: halos, shapes - Galaxy: halo, dynamics, structure

I INTRODUCTION

The relevance of measuring the shapes of dark-matter halos lies in the fact that they are sensitive to the nature of dark-matter particles themselves. Depending on whether these are cold, self-interacting or hot, the shape of a dark halo may be flattened (e.g. Dubinski & Carlberg 1991), close to spherical (Yoshida et al. 2000; Davé et al. 2001) or spherical (Mayer et al. 2002). If dark-matter was baryonic, then the halos are expected to be strongly flattened, with a ratio of minor to major axis of about 0.2 (Pfenninger & Combes 1994). If, on the other hand, the observed flat rotation curves are due to a modified law of gravity, i.e. MOND, then the "fictitious dark component" ought to be equivalent to a spherical (massless) halo (Milgrom 2001).

Predictions for the shapes of dark-matter halos come mostly from N-body simulations, although some analytic estimates using ellipsoidal collapse models are also available (Eisenstein & Loeb, 1995). Numerical studies using the Standard CDM power spectrum have often yielded triaxial halos with minor-to-major axis ratios of $c/a \sim 0.4-0.6$, with cluster size halos more flattened than galaxy size halos (Frenk et al. 1988; Warren et al. 1992; Thomas et al. 1998). More recent studies in a Λ CDM cosmology, have shown the typical axis ratios to be $\sim 0.6-0.8$ for Milky Way size halos (Bullock 2002).

Observationally, the measurement of the shape of a dark-matter halo is a non-trivial task, and as such different techniques using a variety of tracers have found notably disparate values (see Sackett 1999 for a good review). Powerful probes of the shape of dark halos are tidal streams, since they consist of stars moving on very similar orbits (Lynden-Bell & Lynden-Bell 1995; Johnston 1998). For example, the

plane of motion of a tidal stream moving in a spherical potential remains constant in time, implying that its stars will be located on a great circle arc on the sky. Also the width of such a stream will remain constant in an average sense; it will, however, vary periodically depending on the location of its stars along their orbit (Helmi et al. in preparation).

In the case of the Milky Way, Ibata et al. (2001) have concluded that the recently discovered streams from the Sagittarius dwarf galaxy, favor a spherical dark halo, $c/a \sim 1$, with oblate halos with flattening values of $c/a \leq 0.7$ rejected at very high confidence levels (less than 1 chance in one million). Mayer et al. (2002) have also claimed that the mere existence of tidal features around the Milky Way and other nearby galaxies rules out strongly aespherical $(c/a \leq 0.8)$ and triaxial halos, since such features would disperse very rapidly in flattened systems. More recently, Majewski et al. (2003), using M giants from the 2MASS data were able to trace the debris from the Sgr dwarf along an almost complete great circle arc on the sky. The narrowness of the stream and its confinement also led them to suggest a nearly perfectly spherical halo for the Milky Way.

One should, however, bare in mind that most of the Sgr debris discovered thus far is probably constituted by material lost from the dwarf galaxy one to three passages ago, i.e. 1 to 3 Gyr ago, as suggested by the models by Helmi & White (2001) and Martinez-Delgado et al. (2004). As will be shown below such dynamically young debris streams may not be as sensitive to the shape of our Galaxy's dark halo as previously thought.

In this letter, numerical simulations are used to model the evolution of the Sgr dwarf galaxy and of its debris as it orbits the Milky Way. The goal is to determine whether the presently available data on the debris from Sgr can constrain the shape of the Galactic halo as strongly as has been suggested in previous work. With this in mind, we perform several experiments in which the Galactic potential includes a dark halo component with different degrees of flattening. These simulations are introduced in the Section 2. The analysis and results of these simulations are presented in Sec. 3, and we conclude with a brief discussion section.

2 THE SIMULATIONS

In our numerical simulations, we represent the Galaxy by a fixed potential with three components: a dark logarithmic halo

$$\Phi_{\text{halo}} = v_{\text{halo}}^2 \ln(R^2 + z^2/q^2 + d^2), \tag{1}$$

a Miyamoto-Nagai disk

$$\Phi_{\rm disk} = -\frac{GM_{\rm disk}}{\sqrt{R^2 + (a_{\rm d} + \sqrt{z^2 + b_{\rm d}^2})^2}},$$
(2)

and a spherical Hernquist bulge

$$\Phi_{\text{bulge}} = -\frac{GM_{\text{bulge}}}{r + c_{\text{b}}},\tag{3}$$

where d=12 kpc and $v_{\rm halo}=131.5$ km s⁻¹; $M_{\rm disk}=10^{11}\,{\rm M}_{\odot}$, $a_{\rm d}=6.5$ kpc and $b_{\rm d}=0.26$ kpc; $M_{\rm bulge}=3.4\times10^{10}\,{\rm M}_{\odot}$ and $c_{\rm b}=0.7$ kpc. This choice of parameters gives a flat rotation curve with an asymptotic circular velocity of 186 km s⁻¹along the mayor axis of the galaxy, and a circular velocity at the location of the Sun of 229 km s⁻¹, as shown in the top panel of Fig.1. The parameter q is allowed to vary from 0.8 to 1.25, that is, from an oblate to a prolate configuration. In the bottom panel of Figure 1 we show the variation of the density flattening for the halo component in these models. Here q_{ρ} is defined by the ratio of the distances down the z and R axes at which a given isodensity surface cuts those axes (see Binney & Tremaine, 1987, p. 48).

The orbit of Sagittarius is relatively well constrained (Ibata et al. 1997). The heliocentric distance $d \sim 25 \pm 2$ kpc, position $(l, b) = (5.6^{\circ}, -14^{\circ})$, and the heliocentric radial velocity $v_r^{\rm sun} \sim 140 \pm 2 \text{ km s}^{-1}$ of the galaxy core are well determined. Recent proper motion measurements imply $v_{\beta} \sim 280 \pm 20 \text{ km s}^{-1}$ (Ibata et al. 2001). The strong North-South elongation of the system, as well as its debris, suggest that the orbit should be close to polar. To set up the simulations we generate random orbits consistent with these constraints. We also require that the present-day inclination of the orbit defined as L_z/L (the ratio of the z-component to the total angular momentum) does not deviate by more than 5% from that of a plane defined by the cross product of the current position of Sgr \mathbf{r}_{Sgr} and the northern \mathbf{r}_{NS} (and southern r_{SS}) clumps of debris detected by the SDSS (Ivezic et al. 2000: Yanny et al. 2000)

We represent the Sgr galaxy by a collection of 5×10^4 particles and model their self-gravity by a multipole expansion of the internal potential to fourth order (White 1983; Zaritsky & White 1988; see also Helmi & White 2001). For the stellar distribution of the pre-disruption dwarf a King model is chosen, since this has been shown to fit well the dwarf spheroidals satellites of the Milky Way (Mateo 1998). This model is specified by a combination of three parameters: the depth of the potential well of the system $\Psi(r=0)$, a

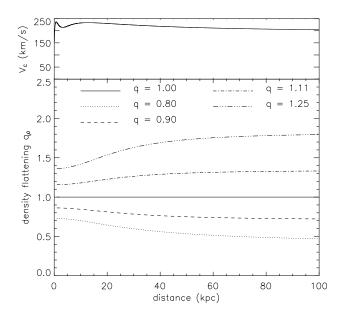


Figure 1. Circular velocity along the major axis of our Galaxy model (top panel). Variation of the dark halo flattening, measured by the density axis ratio, as a function of distance from the centre of our Galaxy model (bottom panel).

measure of the central velocity dispersion σ^2 , and the central density ρ_0 or the King radius r_0 .

In summary, we proceed as follows. For the spherical halo case, we select an orbit as described above, integrate this orbit backward in time (until $t_i = -10$ Gyr), to provide the initial position and velocity of the centre of mass of the satellite in the numerical simulation. We then probe the parameter space of initial models (we leave $\Psi(r=0)/\sigma^2 =$ 3.3 and $r_0 = 0.5$ kpc fixed), until we find one that reproduces currently available observations of the main body and of the debris from Sgr. It turns out that this initial model is close to the purely stellar model of Helmi & White (2001). For the oblate and prolate halo potentials, we select orbits which have similar mean apocentric and pericentric distances, as well as similar L_z , to be able to determine what the effect of varying q is in a direct manner. The initial models for the satellite are chosen so that, after 10 Gyr of evolution, the system has reached a similar degree of disruption as in the spherical halo case, which is typically set to 10 - 20 \%. The orbital properties and models of the satellite are listed in Table 1.

The set of stringent conditions imposed on the orbits and initial models of the satellite are required for a fair comparison of the effect of oblate and prolate halos on the properties of its debris. Note that the aim is not so much to find the best fit model of the Sgr dwarf and of its orbit for each one of the given potentials of the Milky Way, but to compare similar orbits and models to get a handle on how the variation in the degree of flattening affects their evolution. The models presented here do reproduce the properties of the main body of Sgr, as well as the location of its debris. This implies that variations in the characteristics of the debris streams may only be attributed to the variation in the shape of the dark halo component.

q	$\langle q_{ ho} \rangle$	$\langle R_{apo} \rangle$	$\langle R_{peri} \rangle$	L_z	σ	$M~(10^8 M_{\odot})$
0.80	0.60	59.2	10.1	-1105.6	38	6.89
0.90	0.78	56.4	13.7	-911.9	29	4.01
1.00	1.00	53.9	14.5	-938.5	28	3.74
1.11	1.26	51.4	14.0	-929.0	27.5	3.61
1.25	1.63	52.3	14.7	-967	25	2.98

Table 1. Characteristic parameters of the orbits and initial models of Sgr for the different degrees of flattening of the Galactic halo assumed in our simulations. The second column is the mean flattening of the dark halo component within the region probed by the orbit of Sgr. Distances are in kpc, and velocities in km s⁻¹. In all models we have fixed $\Psi(r=0)/\sigma^2=3.3$ and $r_0=0.5$ kpc.

3 RESULTS

The panels in Figure 2 show the sky distribution in equatorial coordinates (α, δ) for all the particles in the different experiments. The particles are colour-coded according to when they became unbound from their parent satellite. Dark grey corresponds to particles unbound in the first 3.5 Gyr of evolution (approximately 5 pericentric passages), light grey to those released between 3.5 and 7 Gyr (that is, between approximately 5 and 9 passages), while black are those that have been released in the last 3 Gyr of evolution (or less than 4 passages ago). This last subset also includes particles that are still bound, i.e. what we call the main body of Sgr.

It immediately strikes the eye that the distribution of black particles is very similar in all five cases. The particles define relatively narrow structures on the sky. These streams are so young dynamically that they have not really been able to experience the flattening of the dark halo in which they orbit, which eventually will lead to their thickening. Therefore, such young streams are not sensitive enough to measure nor constrain the shape of a halo.

The precession and thickening effects, are on the other hand, visible for the oldest streams in the most oblate halo, with q=0.8 and an average density flattening within the region probed by Sgr orbit of $\langle q_{\rho} \rangle = 0.6$. Interestingly, in the case of the prolate halo of similar flattening (1/q=0.8 and $\langle q_{\rho} \rangle = 1.63)$, the oldest streams are much more coherent. This is probably due to the fact that the orbit of Sgr has a high-inclination, i.e. it is essentially aligned with the principal axis of the density distribution, and hence is less subject to strong precession.

For comparison, the first panel of Figure 2 shows the distribution of claimed detections of debris from Sgr discovered so far by the SDSS (Ivezic et al. 2000; Yanny et al. 2000; Newberg et al. 2002) and by Majewski et al. (2003). There seems to be a very good correspondence between the location of this debris and that of the most recently lost particles in our simulations (black dots in the remaining panels). Therefore, our results would seem to imply that the observed streams could be, in fact, made up of stars lost by the Sgr dwarf in the last 3 Gyr of evolution. If this were the case, the immediate conclusion would be that such observations cannot put meaningful constraints on the shape of the Galactic halo.

But are the black dots in Figure 2 really tracing all the debris that has been discovered thus far? To determine

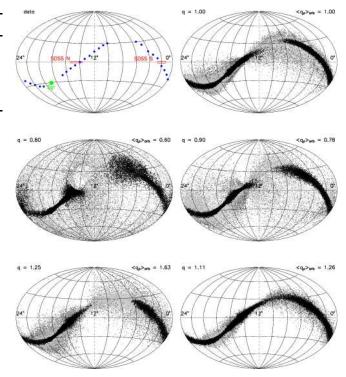


Figure 2. The different panels show the sky distribution (α, δ) of particles in our numerical simulations with varying degrees of flattening for the dark halo of the Galaxy. The different colours represent sets of particles which have become unbound more than 6.5 Gyr ago (dark grey), more than 3 Gyr but less than 6.5 Gyr ago (light grey), and less than 3 Gyr ago (black). In the top left panel, the different detections of debris from Sgr are shown. Here the black solid dots represent approximately the 2MASS M giants stream (Majewski et al. 2003), while the boxes correspond to the northern and southern detections by SDSS (Ivezic et al. 2000; Yanny et al. 2000), which overlap substantially with the detections of the Spaghetti Project Survey (Dohm-Palmer et al. 2001), QUEST (Vivas et al. 2001), Martinez-Delgado et al. (2001), and Dinescu et al. (2002), which are not plotted here for clarity. Direct comparison of the first panel with any of our numerical experiments suggests that the observations could be tracing only the most recently formed streams of Sgr (black dots in the different panels). See Figure 3 for direct evidence that this is indeed the case. Such streams are too young dynamically to have experienced the effects of a flattened halo, be it prolate or oblate, and therefore remain quite coherent even when the minor-to-major density axis ratios c/a are as low as 0.6 within the region probed by the orbit of Sgr, like in the 3rd., $\langle q_{\rho} \rangle = 0.6$, and 5th., $\langle q_{\rho} \rangle = 1.63$, panels of this Figure.

whether this is the case, we also need to check that their dynamical properties (radial velocities and spatial location) are indeed similar. To that end, in Figure 3 we show the distance vs. right ascension distribution of particles for two of our experiments: the most oblate and the most prolate cases. We have used the same colour coding as before, and also overplotted the detections of the northern and southern Sgr debris discovered by SDSS. It becomes now clear that these detections are all tracing only the material that has been released by the Sgr dwarf in the last 3 Gyr. All our experiments of different flattening essentially show the same features, and are in agreement with the observational data available about Sgr and its streams.

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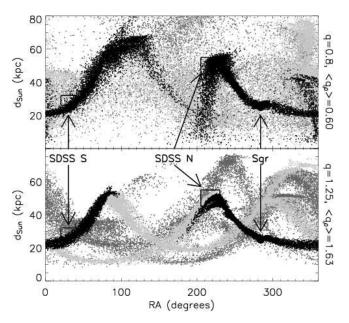


Figure 3. The right ascension vs distance from the Sun distribution of particles in our numerical simulations with q=0.8 (most oblate; top panel) and 1/q=0.8 (most prolate; bottom panel). The same colour coding as in Fig.2 is used. Also shown are the northern and southern detections of debris from Sgr by SDSS. Note that these overlap spatially (in the sky and in distance, as well as in radial velocity) with the most recently unbound material, indicated here by the black solid dots. This confirms that the streams discovered thus far are dynamically young.

We also note here that the tidal streams traced by the 2MASS M giants in Majewski et al. (2003), are also well-represented by the black dots shown in Fig. 2 and Fig. 3.

The light grey dots representing particles released between 3.5 and 7 Gyr ago, are distributed along a thicker great circle path on the sky in all but the most oblate halo experiments. Their sky location overlaps substantially with the dynamically young streams and with the observations of the streams from Sgr detected thus far. However, when the additional dynamical information that is available is used, it becomes clear that the overlap is only due to a projection effect, and that these intermediate age streams have a different 3-dimensional spatial distribution, as evidenced by their different distance distribution as shown in Fig. 3. There are currently no observational data that would correspond to these intermediate age streams, although see Dohm-Palmer et al. (2001) for a possible hint.

4 DISCUSSION

We have carried out numerical experiments of the evolution of the Sgr dwarf and its debris orbiting in Galactic potentials with dark halo components with different degrees of flattening. These experiments show that dynamically young streams (younger than about 3 Gyr) on a Sgr-like orbit remain very coherent spatially, defining great circle arcs on the sky even when the mean minor-to-major density axis ratio c/a of the dark halo of our Galaxy model within the region probed by Sagittarius orbit is as low as 0.6 (prolate and oblate cases). A straightforward comparison to the various

detections of debris material from Sgr leads to the unavoidable conclusion that this debris is dynamically young. Hence its striking confinement to a great circle arc on the sky does not imply that the dark halo of our Galaxy is nearly spherical. This is in agreement with the conclusions of Martinez-Delgado et al. (2004).

These results would seem to be in contradiction with those of Ibata et al. (2001). The difference probably lies in the fact that, in their simulations, the dwarf is completely disrupted by the present time (no core can be seen in any of the figures corresponding to flattened mass distributions). Hence they are unable to distinguish streams as dynamically young as ours. This can either be due to the fact that the chosen orbits were not a good representation of the actual orbit of Sgr, or to their initial model of the dwarf, upon which they probably did not impose any restrictions concerning its survival.

It is perhaps worth emphasizing that to produce similar end products (i.e. main body of the Sgr dwarf) in simulations with different potential shapes, adjustments in the initial model of the dwarf are necessary. For comparison, Ibata et al. (2001) use only two models in their experiments. These are the models that evolved in a spherical potential reproduce the properties of the main body (these were presented in the papers by Ibata & Lewis 1998; and Helmi & White 2001). When these initial models of the Sgr dwarf were evolved in potentials with different degrees of flattening, they gave rise to quite different present-day dwarfs: the evolution of the dwarf galaxy is not independent of the potential in which it orbits. In contrast to the approach adopted here, Ibata et al. did not "correct" their models for this effect and this is why their dwarf in the flattened potentials does not survive until the present time, nor produces dynamically young streams that would be visible today.

It becomes clear that the initial conditions imposed on the orbit as well as on the model of the progenitor of Sagittarius are both critical to the interpretation of the numerical experiments run under different dark matter halo shapes. Is it possible to constrain them in a realistic way? Concerning the initial models of the dwarf, observations of the main body of Sgr can be used as boundary conditions. It is desirable that the observed characteristics of Sgr are reproduced before one addresses the properties of its debris. All models discussed in this paper satisfy this requirement in the following sense. They reproduce the properties of the main body described by Ibata et al. (1997), in terms of the extent, surface brightness and internal velocity dispersion. A good correspondence is also found for the number counts and the contrast between the main body and the M giant streams. This is shown in Fig.4, and should be compared to Fig.13 of Majewski et al. (2003).

The main difference between the stellar model discussed in Helmi & White (2001) and the models presented here, lies in the orbital parameters (slightly larger apocentre and pericentre distances; these were less well constrained three years ago). The mass ratio of the remnant to the initial dwarf ranges between 10-20% in all cases. The fraction of mass contained in the dynamically youngest streams in the stellar model of Helmi & White (2001) is $\sim 10\%$, while for the models presented in this paper it varies from 13% to 25% of the initial mass of the system. This implies slightly

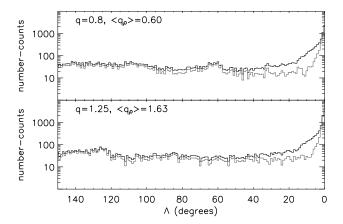


Figure 4. Number counts of particles located within 5 kpc of the plane defined by the Sgr dynamically young debris, as a function of angular distance along the trailing stream Λ (see for more details Majewski et al. (2003)). The grey histogram corresponds to the actual number counts, while the black curve includes the effect of a background as measured by Majewski et al. (2003). There is good agreement between the black histograms (both for q=0.8 and q=1.25) and the data shown in Fig.13 of Majewski et al. (2003). However, the model dwarf (background subtracted) appears to be more concentrated than Majewski et al. (2003) suggest, although its characteristics are consistent with previous observations of Sgr.

different recent mass-loss rates for each one of the models presented here. However, the similarity in the spatial distribution of the youngest streams in all our experiments indicates that this is not strongly dependent on the detailed recent mass-loss history. The spatial distribution of stars in these dynamically young streams is mostly determined by the recent orbital trajectory, which by construction is similar in all cases (see also Martinez-Delgado et al. 2004).

We have chosen to model the dwarf using a King profile with only one component of stellar nature. We have not taken into account the effect of a dark-matter halo encompassing the stellar distribution, which would have been important during the initial stages of evolution of the dwarf (see Jiang & Binney 2000). However, it is very likely that the main body of the present day Sagittarius represents only the central region of the progenitor dwarf (Helmi & White 2001); and we may assume that the dominant component of this central region is stellar. This implies that the properties of the streams formed in the last pericentric passages, i.e. those that are dynamically young, will not to differ substantially from those originated in dark-matter dominated progenitors. To support this statement, in Fig. 5 we have plotted the distribution of "star-like" particles for the dark-matter dominated model of Helmi & White (2001). The stellar component of this model initially is very similar to the purely stellar model discussed in the same paper (see their Fig. 6). The mass-to-light ratio varies with radius (more precisely with binding energy), implying that the streams formed at early times are dark-matter dominated, while those dynamically young are purely stellar.

The left panel of Fig.5 shows the sky distribution of "dark-matter" (in grey) and of "star" particles (in black). The panel on the right shows their right ascension vs distance distribution (same colour coding). These plots should

be compared to Figs.2 and 3. We note that the dynamically young streams produced in a dark-matter dominated system, whose initial stellar distribution is similar to that of the purely stellar systems discussed in this paper, are not significantly different to those produced in purely stellar systems. This is the case because the streams highlighted here, and which we believe correspond to the streams observed, are formed by material lost in the last few pericentric passages and originate in regions of the initial system which were dominated by stars (rather than by dark-matter). The situation would perhaps be different if the Sgr dwarf was a system in dynamical equilibrium, currently surrounded by a massive dark halo. The most recently formed streams, would then have a large-dark matter content. If however, as most authors conclude (Johnston, Spergel & Hernquist 1995; Velazquez & White 1995; Zhao 1998; Gómez-Flechoso, Fux & Martinet 1999; Helmi & White 2001; Majewski et al. 2003), Sgr is close to complete disruption, then the youngest streams should be made up of material which is star dominated, as in these simulations. The high mass-to-light ratios suggested by the observations are most likely due to the fact that the system is not in virial equilibrium, and its large internal velocity dispersion is inflated due to the effect of tides, which are strongest close to pericentre.

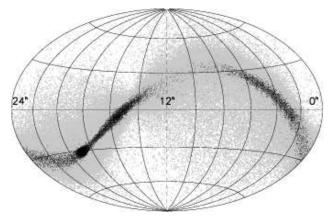
To measure the shape of the dark halo of the Galaxy reliably dynamically old streams are required, and this implies using stars that trace old stellar populations, such as RR Lyrae stars, the BHBs and the red giants, as well as main sequence turn-off stars. Such older streams will typically have very low surface brightness, and hence are more difficult to detect, particularly if the dark halo of our Galaxy is not spherical.

While this paper was being written, Newberg et al. (2003) found an excess of BHB stars at a distance of 80 kpc from the Sun, in a direction which lies 10 kpc away from the plane defined by the debris of Sgr. This prompted them to suggest that it could be material lost by the dwarf in earlier passages. Notably, Fig. 3 of this Letter shows, for our simulations of the most oblate and most prolate halos, the presence of debris at a similar location in the sky $(110^{\circ} < \alpha < 130^{\circ})$ and $\delta \sim 30^{\circ}$) and at a similarly large distance. This debris is old (lost in the first 3 passages) yet it is not far from the great circle defined by the young debris streams. Thus, even though the Newberg et al. (2003) result seems at first sight to be very compelling, more careful modeling and a better estimate of the width of the stream, as well as velocity data on the BHB star candidates are needed to really constrain the shape of the dark halo of the Galaxy. Interestingly the presence of such ancient streams necessarily imply that Sgr has already been orbiting the Galaxy for about 10 Gyr.

No other prominent stellar streams have been found thus far in the Galactic halo. Although the volume probed by the surveys to date has been somewhat limited, the situation is changing rapidly. The lack of bright and spatially coherent stellar streams in the Galactic halo could be suggesting that this is in fact rather flattened. On the other hand, it could be indicative of a low satellite accretion rate or a hint that the majority of the satellites accreted are dark. It is important to disentangle these issues; they provide direct constraints both on the nature of dark-matter as well as on the process of galaxy formation.

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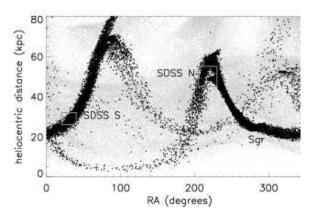


Figure 5. The sky and right ascension vs distance from the Sun distribution of particles in the "stars + dark-matter" model of Helmi & White (2001). Stars are shown in black, while dark-matter particles are grey. Even in a dark-matter dominated system like that shown here, there is a good correspondence between the properties of the streams observed and those that are dynamically the youngest.

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